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Misinterpretation yields supervelocities during transmission of wave packets through a barrier

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Abstract. This paper is concerned with the transmission time of an incident Gaussian wave packet through a symmetric rectangular barrier. Following Hartman (J. Appl. Phys. **33**, 3427 (1962)), the transmission time $\tau_{\rm Ha}$ is usually taken as the difference between the time at which the peak of the transmitted packet leaves the barrier of thickness ℓ and the time at which the peak of the incident Gaussian wave packet arrives at the barrier. This yields a corresponding transmission velocity $c_{\rm Ha} = \ell/\tau_{\rm Ha}$ which appears under certain conditions as a supervelocity, *i.e.* becomes larger than the corresponding propagation velocity in free space which is the group velocity for electrons or the velocity of light for photons, respectively. By analysing the propagation of a broadband wave packet (which leads in free space to an extremely concentrated wave packet at a certain time) we obtain the pulse response function of the barrier and show that the insertion of the barrier is physically unable to produce a supervelocity. Therefore, the peak of an incident Gaussian wave packet and the peak of the transmitted wave packet are in no causal relationship. The shape of the transmitted wave packet is produced from the incident wave by convolution with the pulse response of the barrier. This yields a distortion of the shape of the wave packet which includes also the observed negative time shift of the peak. We demonstrate further that the phenomenon of Hartman's supervelocities is not restricted to barriers with their exponentially decaying fields but occurs for instance also in transmission lines with an inserted LCR circuit.

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1 Introduction

Time-dependent tunneling of particles through a barrier was studied for the first time by MacColl [1] in 1932 by means of an incident Gaussian wave packet. He concluded from his simplified treatment that there is no appreciable delay in the transmission of a wave packet through a barrier.

In order to estimate the transmission times for metal-insulator-metal tunneling junctions, Hartman [2] treated in 1962 this problem again and in more detail. He avoided the numerical integration of the derived definite integral for the transmitted wave packet and applied instead the *method of stationary phase* to the integrand. This method allows to determine the instantaneous location of the constructive interference of the Fourier components of the wave packet and, hence, the instantaneous location of the peak of the moving wave packet. If one assumes a very narrow k distribution with its center at the wave vector k', a very broad real space wave packet results showing a nearly monoenergetic particle energy $E = E(k')$. In addition, the method of stationary phase reveals (see Sect. 2.3) that the envelope of the packet propagates with a group velocity which is, in the one-dimensional case,

$$c_{\rm gr}(k') = \frac{1}{\hbar}\partial E(k)/\partial k\Big|_{k=k'}. \qquad (1)$$

The group velocity was introduced by Lord Rayleigh [3] in 1877 and the method of stationary phase by Thomson [4] in 1887. Both studied wave propagation in shallow water and also other classical waves with dispersion.

Hartman used the method of stationary phase to calculate the development in time and the position of the peak of the propagating wave packets before and also after the barrier. There is no objection to this procedure. However, we will raise objection to his next step. He defined the transmission time $\tau_{\rm Ha}(E)$ as the difference between the time at which the peak of the transmitted packet leaves the barrier and the time at which the peak of the incident nearly monoenergetic Gaussian wave packet arrives at the barrier. Nowadays, this time difference is often called the *phase-delay time*. For definition and discussion of other transmission times in use, see the reviews of Hauge and Støvneng [5], Olkhovsky and Recami [6], and Landauer

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and Martin [7]. Hartman derived (see Sect. 2.3) an analytic expression for $\tau_{\rm Ha}(E)$ and compared the numerical results in a graph with the "vacuum transmission time" $\tau_{\rm vac}(E) = \ell/v_{\rm gr}(E)$ which is the time required for an incident nearly monochromatic Gaussian wave packet of energy E to transverse a distance equal to the barrier thickness ℓ. Under certain conditions, $\tau_{\rm Ha}(E)$ came out to be shorter than $\tau_{\rm vac}(E)$ and saturates to a constant value which is independent of ℓ, *i.e.* a *supervelocity* seemed to occur for the particle passing through the barrier in comparison with the group velocity $c_{\rm gr}(E)$ for the particle propagating in free space. At that time in 1962, this result was acceptable, since it seemed not to violate fundamental principles of physics.

However, there exists a strong analogy between particle tunneling through a potential barrier and the transmission of an electromagnetic wave or photon through the air gap between two prisms under conditions of total reflection [8]. In order to study photon tunneling in analogy to electron tunneling, Martin and Landauer [9] proposed in 1992 a suitable setup for electromagnetic waves using a rectangular metal waveguide filled with a material of dielectric constant $\epsilon_1 > 1$. Only a barrier region of thickness ℓ contains air ($\epsilon = 1$). The frequencies are chosen in such a way that transmission occurs in the dielectric filled waveguide but exponentially decay in the air gap. For this waveguide setup, Martin and Landauer derived the same analytic expressions for the complex (amplitude) transmission factor as it is known from Schrödinger's equation for a rectangular barrier. Thus, Hartman's method can be transferred directly to this waveguide setup and predicts the occurrence of a supervelocity for passing the barrier. Already in 1983, Bosonac [10] found theoretically that an electromagnetic wave packet is transferred with superluminal velocity across a vacuum barrier under conditions of total reflection if the peaks of the wave packets are taken as reference. In this case, the problem occurs that this supervelocity is higher than the velocity c of a plane wave of light in vacuum.

Einstein [11] founded in 1905 his theory of special relativity on the principle of relativity ("physical laws are the same in all inertial systems") and on the principle that "the highest velocity of light in each inertial system is c". By adding the principle of (strict) causality ("no effect can precede the cause"), Einstein [12] showed in 1907 that, within the validity of these three principles, c is the highest velocity for all kinds of *signals that are able to produce a physical effect*. To our present knowledge, these signal events can only be produced by mass or energy transfer. Other often discussed and only mathematically as a function of time defined "signals" may have no physical relevance. Special waveforms (*i.e.* plane wave, exponentially decaying near field, dipole wave, or others) are irrelevant in Einstein's derivation. Therefore, we have to conclude that the occurrence of a superluminal velocity in the exponentially decaying near field of the barrier violates at least one of the three cited principles. However, for this statement, one has to verify that the superluminal velocity under question is actually a velocity of matter or energy, *i.e.* it is a causal velocity which only can produce a physical effect.

Recently, several *waveguide experiments* [13,14] have been done which show with certainty that the transmission time can be so short that superluminal velocities occur – whereby however, the transmission time was measured according to the stated definition of Hartman using the peaks of the two wave packets. Also, optical experiments, using two prisms under conditions of total reflection with a variable air gap between them [15] or using a stopband of dielectric multilayers [16,17], are in accordance with Hartman's superluminal velocities.

Büttiker and Landauer [18] suggested in 1982 that there is no physical justification for connecting the incident peak to the transmitted peak. Landauer and Martin [19] tried in 1992 to demonstrate this lack of causality between both peaks by discussing a special model. They stated that the high-energy components of a Gaussian particle wave propagate faster than the other components and concluded that they therefore reach the barrier first. They argued further: since the high-energy components are more effectively transmitted than the following components of lower energy, the transmitted wave packet is mainly due to the front part of the incident wave packet and hence both peaks are not causally related. However, one has to object that a position within the wave packet for high-energy and low-energy particles does not exist since they are described as Fourier components. Moreover, it is obvious that such a model fails to explain the electromagnetic case, where dispersion can be avoided.

Yun-ping and Dian-lin [20] explained in 1995 how an apparent superluminal velocity can occur due to reshaping a photonic wave packet as a consequence of destructive interference in a multi-path setup.

In the course of the following discussion of electron and photon tunneling through a barrier, it will become evident too that there is indeed no causal relationship between both peaks. The positive or negative time delay for the peak of the transmitted wave packet is influenced by both the phase change and the amplitude reduction of the plane wave components in passing the barrier or any other obstacle.

We start in Section 2 with the quantitative description of the transmission of incident plane waves and Gaussian wave packets through a barrier and explain Hartman's treatment which leads to supervelocities. In Section 3, we will prove that the insertion of the barrier never produces a shorter *physical* transmission time than those that occurs without barrier, *i.e.* no *physical* supervelocities occur. Thereafter, in Section 4, we will show that the distortion of the wave packet and the associated negative time shift of the peak position can also be observed with a LCR-resonance circuit inserted in a double-conductor transmission line. Other suitable circuits can also be used. Here, no barrier or tunneling is involved. As a consequence of these investigations, we will have to conclude that Hartman's definition of the transmission velocity and the corresponding measured transmission "velocities" are irrelevant from the physical point of view.

2 Transmission of incident plane waves and Gaussian wave packets through a barrier

For the quantitative discussion, in the following sections we present the necessary analytic expressions and perform numerical calculations in order to illustrate and better understand the present problem. In discussing Hartman's method, we use also a slightly modified notation and use a unified treatment for particle energies E above and below the barrier energy V_1. Figure 1 may help to explain the notation and the procedure.

2.1 Rectangular barrier as a linear time-invariant system

We start with a particle travelling in the $+z$ direction using the time-dependent plane-wave solution

$$\psi_k(z,t) = \psi_0 \mathrm{e}^{\mathrm{i}\{kz - E(k)t/\hbar\}} \tag{2}$$

of Schrödinger's wave equation, which connects the one-particle energy E with the wave vector k by the dispersion relation

$$E(k) = \hbar^2 k^2/2m + V \quad \text{where} \quad k = \sqrt{2m\{E - V\}}/\hbar, \tag{3}$$

where m is the effective mass and V is a constant potential. We treat a symmetric rectangular barrier and take the potential $V = 0$ in the region (0) before and in the region (2) after the barrier, and $V = V_1 > 0$ in the barrier region (1) (see Fig. 1). To avoid in the following the superscripts in the corresponding three wave vectors $k^{(0)}$, $k^{(1)}$, and $k^{(2)}$, we will use the notation $k \equiv k^{(0)} = k^{(2)}$ in both halfspaces with $V = 0$ and replace later in equation (16) the quantity $\mathrm{i}k^{(1)}$ in the barrier region ($V = V_1$) by the propagation constant γ.

In order to describe the incident wave packet $\psi(z,t)$, a suitable superposition of these plane waves is chosen. This corresponds to the Fourier integral

$$\psi(z,t) = \int_{-\infty}^{\infty} \underline{\psi}(k,t)\mathrm{e}^{\mathrm{i}kz}\mathrm{d}k/2\pi, \tag{4}$$

where the Fourier components in z at time t are given by

$$\begin{aligned}\underline{\psi}(k,t) &= \underline{\psi}(k,t=0)\mathrm{e}^{-\mathrm{i}E(k)t/\hbar} \\ &= \underline{\psi}(k,t=0)\mathrm{e}^{-\mathrm{i}\hbar k^2 t/2m}\end{aligned} \tag{5}$$

with $\underline{\psi}(k,t=0)$ the spectral probability amplitude of the plane wave components which have to be specified at $t = 0$. Only spectral components with positive wave vector $k > 0$ can contribute to a wave packet propagating in the $+z$ direction.

Since we have a constant potential in each of the three regions, we are concerned with a *linear time-invariant* system. In a linear system, the superposition of system inputs leads to an output which consists of a superposition of the partial outputs which have the same weights as the partial inputs. In our case, the partial system inputs are the plane waves (described by Eq. (2)) which are connected with their outputs – their transmitted plane waves (stationary solutions) $\psi_k^T(z,t)$ – by the complex (amplitude) transmission factor $\underline{T}(k)$

$$\psi_k^T(z,t) = \underline{T}(k)\psi_k(z,t). \tag{6}$$

Therefore, the whole input (Eq. (4)) generates as an output the transmitted wave packet valid for z after the barrier

$$\psi^T(z,t) = \int_{-\infty}^{\infty} \underline{T}(k)\underline{\psi}(k,t)\mathrm{e}^{\mathrm{i}kz}\mathrm{d}k/2\pi. \tag{7}$$

Applying the well-known convolution theorem, the transmitted wave packet can also be expressed as a convolution $T(z) * \psi(z,t)$ valid for z after the barrier

$$\begin{aligned}\psi^T(z,t) &= \int_{-\infty}^{\infty} T(z-z')\psi(z',t)\mathrm{d}z' \\ &= \int_{-\infty}^{\infty} T(z')\psi(z-z',t)\mathrm{d}z',\end{aligned} \tag{8}$$

where we have used the Fourier integral (4) and

$$T(z) = \int_{-\infty}^{\infty} \underline{T}(k)\mathrm{e}^{\mathrm{i}kz}\mathrm{d}k/2\pi. \tag{9}$$

2.2 Scattering of plane waves: (amplitude) transmission factor

As shown in Figure 1, we denote the barrier thickness by ℓ and the height by V_1. Instead of V_1, we may also use the corresponding wave vector k_V, defined by

$$E(k_V) = V_1 \quad \text{or} \quad k_V = \sqrt{2mV_1}/\hbar. \tag{10}$$

The transmission of plane waves through a rectangular barrier was first investigated by Nordheim [21] in 1928 and a sign error was corrected by Fowler [22] in 1929. The (amplitude) transmission factor can be written [23] as

$$\underline{T}(k) = \frac{\mathrm{e}^{-\mathrm{i}k\ell}}{\cosh\gamma\ell + \frac{\mathrm{i}}{2}\{\gamma/k - k/\gamma\}\sinh\gamma\ell}, \tag{11}$$

where the propagation constant $\gamma \equiv \gamma^{(1)}$ in the barrier region (1) is expressed by

$$\gamma^2 = k_V^2 - k^2 \quad \text{or} \quad \gamma = \mathrm{i}k\sqrt{1 - k_V^2/k^2} = \mathrm{i}\sqrt{k^2 - k_V^2}, \tag{12}$$

i.e., we have to choose in the following the sign of the square root always positive in order to obtain the correct propagation constant in the limit $k_V = 0$. Hence, we have

$$\gamma = \begin{cases} \mathrm{i}k^{(1)} = \mathrm{i}\sqrt{k^2 - k_V^2} = \mathrm{i}k_V\sqrt{E/V_1 - 1} & \text{if} E > V_1 \\ -\alpha^{(1)} = -\sqrt{k_V^2 - k^2} = -k_V\sqrt{1 - E/V_1} & \text{if} E < V_1, \end{cases} \tag{13}$$

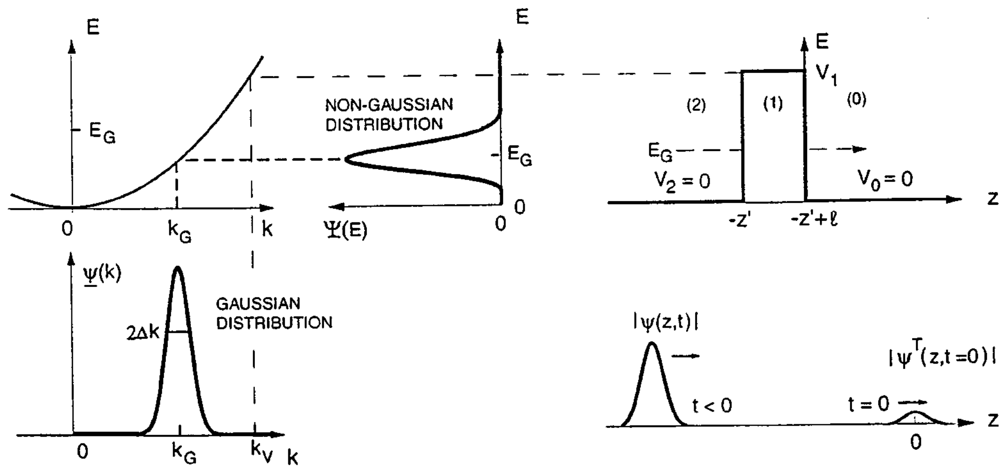

Fig. 1. Sketch of a Gaussian k spectrum, the dispersion curve, and the energy spectrum, the rectangular potential barrier, and the wave packet before and after the barrier.

since $E/V_1 = k^2/k_V^2$.

For all possible values of γ, the first term in the denominator of (11) is always real and the second imaginary. Therefore, we can easily rewrite $\underline{T}(k)$ in polar form by using its absolute value and its phase factor

$$\underline{T}(k) = |\underline{T}(k)|\mathrm{e}^{\mathrm{i}\varphi^T(k)}, \tag{14}$$

where the absolute value is obtained as

$$\begin{aligned} |\underline{T}(k)| &= \frac{1}{\sqrt{\cosh^2\gamma\ell + \frac{1}{4}\{\gamma/k - k/\gamma\}^2\sinh^2\gamma\ell}} \\ &= \frac{1}{\sqrt{1 + \frac{1}{4}\{\gamma/k + k/\gamma\}^2\sinh^2\gamma\ell}} \\ &= \frac{1}{\sqrt{1 + \sinh^2\left(k_V\ell\sqrt{1-E/V_1}\right)/\{4(1-E/V_1)E/V_1\}}} \end{aligned} \tag{15}$$

and where the phase $\varphi^T(k)$ is given by

$$\begin{aligned} \varphi^T(k) &= -k\ell - \arctan\left\{\frac{1}{2}\{\gamma/k - k/\gamma\}\tanh\gamma\ell\right\} \\ &= -k\ell + \arctan\left\{\frac{\sqrt{E/V_1} - \sqrt{V_1/E}/2}{\sqrt{1-E/V_1}} \right. \\ &\qquad \left. \times \tanh(k_V\ell\sqrt{1-E/V_1})\right\}. \end{aligned} \tag{16}$$

As can be seen, we have in the limit $E \gg V_1$ the phase $\varphi^T(k) = 0$ which yields as expected a transmission factor $\underline{T}(k) = 1$. At $E = V_1$, the phase is $\varphi^T(k) \approx -\mathrm{i}k\ell + 30°$.

Figure 2 gives an impression of the absolute value and of the phase of $\underline{T}(k)$ as a function of the two dimensionless parameters k/k_V and $k_V\ell$. The barrier can be characterized by the single parameter $k_V\ell$.

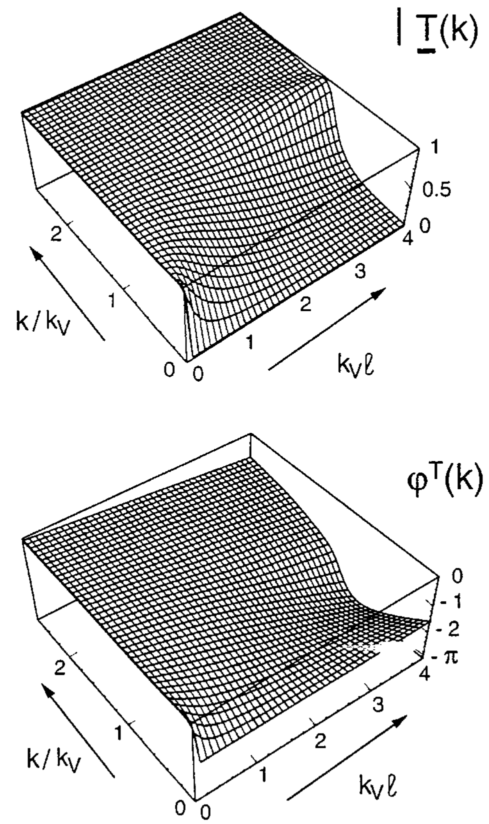

Fig. 2. Graphic representation of the complex (amplitude) transmission factor $\underline{T}(E(k))$ of a symmetric rectangular barrier. The absolute value and phase are plotted as a function of the two dimensionless quantities $k/k_V = \sqrt{E/V_1}$ and $k_V\ell$, where V_1 is the barrier height and ℓ is its thickness.

2.3 Hartman's transmission time and associated supervelocities

The method of stationary phase gives the position z of constructive interference of the Fourier components within the wave packet at each time. Applied to the integrand

$$F(k/k_V, k_V\ell) = \frac{1}{\ell}\frac{\partial}{\partial k}\varphi^T(k) + 1 = \frac{\sinh\left(2k_V\ell\sqrt{1-E/V_1}\right)/\{k_V\ell\sqrt{1-E/V_1}\} + \{1-2E/V_1\}2E/V_1}{\{1-E/V_1\}4E/V_1 + \sinh^2\left(k_V\ell\sqrt{1-E/V_1}\right)}\cdot \tag{21}$$

of (4) with real $\underline{\psi}(k, t=0)$, we obtain for the incident wave packet the condition

$$0 = \frac{\partial}{\partial k}\{kz - E(k)t/\hbar\} = z - \frac{1}{\hbar}\frac{\partial E(k)}{\partial k}t$$
$$= z - c_{\rm gr}(k)t \tag{17}$$

and for the transmitted wave packet according to (7, 14) the condition

$$0 = \frac{\partial}{\partial k}\left\{\varphi^T(k) + kz - E(k)t/\hbar\right\}$$
$$= z - c_{\rm gr}(k)t + \frac{\partial}{\partial k}\varphi^T(k). \tag{18}$$

If $\underline{\psi}(k, t=0)$ is complex, the phase contributions would cancel later in calculating $\tau(k)$. According to (17), the peak of the incident wave packet reaches the barrier at $z = -z'$ at the time $t_1 = -z'/c_{\rm gr}(k)$. At $z = -z' + \ell$, the peak of the transmitted packet appears at the time $t_2 = 1/c_{\rm gr}(k)\{-z' + \ell + \partial\{\varphi_{\underline{T}}(k)\}/\partial k\}$.

Hence, the transmission time due to Hartman is

$$\tau_{\rm Ha}(k) = t_2 - t_1 = \frac{\ell}{c_{\rm gr}(k)}\left\{\frac{1}{\ell}\frac{\partial}{\partial k}\varphi^T(k) + 1\right\}$$
$$\equiv \tau_{\rm vac}(k)F(k/k_V, k_V\ell). \tag{19}$$

Here, we have introduced as an abbreviation the factor $F(k/k_V, k_V\ell)$ which has the meaning of a time delay factor. Its reciprocal value describes the relative enhancement of the mean transfer velocity

$$c_{\rm Ha}(k)/c_{\rm gr}(k) = 1/F(k/k_V, k_V\ell). \tag{20}$$

This factor can be written by means of (16) as

see equation (21) above.

Whereas Hartman was mainly interested in the transmission time, we are more interested in the velocity ratio (20). We have calculated this ratio and obtained the graph of Figure 3. As can be seen, the already mentioned super-velocities $c_{\rm Ha}(k) > c_{\rm gr}(k)$ occur in the tunneling region $k < k_V$ for barriers with $k_V\ell$ about unity and larger. In the limit of a high and broad barrier ($E \ll V_1$, $k_V\ell \gg 1$), we obtain directly from (21) that Hartman's transfer velocity $c_{\rm Ha}(k)/c_{\rm gr}(k)$ becomes $k_V\ell/2$ and thus rises proportional to the barrier length ℓ.

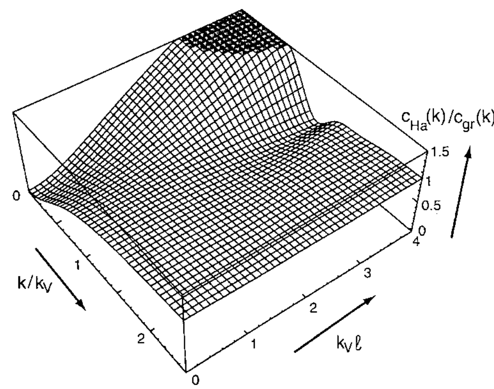

Fig. 3. Graph of the mean transfer velocity $c_{\rm Ha}(k)$ as can be deduced from Hartman's assumptions in comparison with the group velocity $c_{\rm gr}(k)$ which occurs before and after the barrier region.

2.4 Scattering of a Gaussian wave packet

To confirm that the method of stationary phase used by Hartman gives the right result, we consider a Gaussian wave packet. In order to obtain a normalized incident Gaussian pulse in (4), one has to choose a Gaussian distribution in the k spectrum at $t = 0$, which may be centered at $k_{\rm G} > 0$ with the k spread Δk in the form

$$\underline{\psi}(k, t=0) = \sqrt{2\sqrt{\pi}/\Delta k}\,{\rm e}^{-\frac{1}{2}\left\{\frac{k-k_{\rm G}}{\Delta k}\right\}^2}. \tag{22}$$

Introducing this distribution into (4) and performing the integration yields a Gaussian wave packet in real space with a maximum height and with minimum spread at $t = 0$. The wave packet becomes broader and lower with increasing time, but remains always a normalized Gaussian wave packet. As expected, the propagation of the peak occurs with group velocity $c_{\rm gr}(k_{\rm G})$. One finds furthermore that the time-dependent change of the pulse can be neglected for times in the range $\pm\Delta t$ fulfilling the condition

$$c_{\rm gr}(k_{\rm G})\Delta t \ll k_{\rm G}/\{\Delta k\}^2. \tag{23}$$

Under this condition, the propagating Gaussian wave is given by

$$\psi(z,t) = \sqrt{\Delta k/\pi}\,{\rm e}^{{\rm i}\{k_{\rm G}z - E_{\rm G}t/\hbar\}}{\rm e}^{-\{z-c_{\rm gr}(k_{\rm G})t\}^2\Delta k^2/2} \tag{24}$$

where $E_{\rm G} \equiv \hbar^2 k_{\rm G}^2/2m$. The Gaussian envelope (last factor in (24)) propagates with group velocity $c_{\rm gr}(k_{\rm G}) = \hbar k_{\rm G}/m$ and contains an oscillating wave (second factor in (24)) which travels with the slower phase velocity

$$c_{\rm ph}(E_{\rm G}) = \{E_{\rm G}/\hbar\}/k_{\rm G} = \hbar k_{\rm G}/2m. \tag{25}$$

$$\psi^T(z,t) = \sqrt{2\sqrt{\pi}/\Delta k}\int_{-\infty}^{\infty} \underline{T}(k)\mathrm{e}^{-\frac{(k-k_\mathrm{G})^2}{2(\Delta k)^2}}\mathrm{e}^{\mathrm{i}\{kz-E(k)t/\hbar\}}\mathrm{d}k/2\pi$$
$$= 1/\sqrt{2\pi\sqrt{\pi}\Delta k}\mathrm{e}^{\mathrm{i}\{k_G z-E(k_\mathrm{G})t/\hbar\}}\int_{-\infty}^{\infty}\mathrm{d}k\,\underline{T}(k)\mathrm{e}^{-\frac{(k-k_\mathrm{G})^2}{2(\Delta k)^2}}\mathrm{e}^{\mathrm{i}\left\{(k-k_\mathrm{G})z-[c_\mathrm{gr}(k_\mathrm{G})-\frac{\hbar}{2m}(k-k_\mathrm{G})]t\right\}}. \quad (26)$$

If a barrier is inserted, the transmitted wave packet is, according to (7, 22),

see equation (26) above.

Neglecting the pulse broadening due to dispersion, the absolute value of the transmitted wave packet is given by

$$|\psi^T(z,t)| = 1/\sqrt{2\pi\sqrt{\pi}\Delta k}$$
$$\times\left|\int_{-\infty}^{\infty}\mathrm{d}k\frac{\mathrm{e}^{-\mathrm{i}k\ell}}{\cosh\gamma\ell+\frac{\mathrm{i}}{2}\{\gamma/k-k/\gamma\}\sinh\gamma\ell}\right.$$
$$\left.\times\,\mathrm{e}^{-\frac{(k-k_\mathrm{G})^2}{2(\Delta k)^2}}\mathrm{e}^{\mathrm{i}(k-k_G)\{z-c_\mathrm{gr}(k_\mathrm{G})t\}}\right|. \quad (27)$$

It seems to be impossible to obtain an analytic solution of this integral. However, if the energy $E_\mathrm{G} \equiv E(k_\mathrm{G})$ of the peak of the Gaussian distribution lies very high above V_1, we will obtain an unperturbed propagation of the incident Gaussian pulse. By lowering E_G more and more, an increasing part of the lower end of the k distribution will be removed by the barrier. In any case, the remaining k distribution is shifted in phase.

In order to obtain information about the true envelope of the transmitted wave packet, we performed a numerical investigation of relation (27). We choose an incident normalized Gaussian wave packet with a spread $\Delta k/k_\mathrm{G} = 0.1$ and calculated, for three different peak positions in k space ($k_\mathrm{G}/k_V = 0.9$, 0.7, and 0.5) and for a set of twelve $k_V\ell$ values ($k_V\ell = 0$, 1, to 11), the absolute value $|\psi^T(z,t)|$ of the transmitted wave packet. These results are compiled as graphs in Figure 4. The absolute value is given in dimensionless form $|\psi^T(z,t)|/\sqrt{k_V}$ as a function of the dimensionless propagation argument $k_V\{c_\mathrm{gr}(k_\mathrm{G})t - z\}$. For each of the three chosen k_G/k_V positions, the incident Gaussian wave packet is represented by the envelope denoted by $k_V\ell = 0$ since $\ell = 0$ means an unperturbed propagation in the z direction. The width of the Gaussian wave packets rises from $k_\mathrm{G}/k_V = 0.9$ to 0.5 since the relative band width $\Delta k/k_\mathrm{G}$ in k space is taken as a constant. The peak reaches position $z = 0$ at time $t = 0$ and propagates thereafter in the halfspace $z > 0$ with the same group velocity $c_\mathrm{gr}(k_\mathrm{G})$. If we have a barrier of thickness $\ell > 0$ anywhere in the region on the left hand of our observation interval, the eleven envelopes ($k_V\ell = 1$ to 11) represent the transmitted wave packets which can be directly compared with the Gaussian wave packet $k_V\ell = 0$ since they propagate with the same group velocity $c_\mathrm{gr}(k_\mathrm{G})$. According to condition (23),

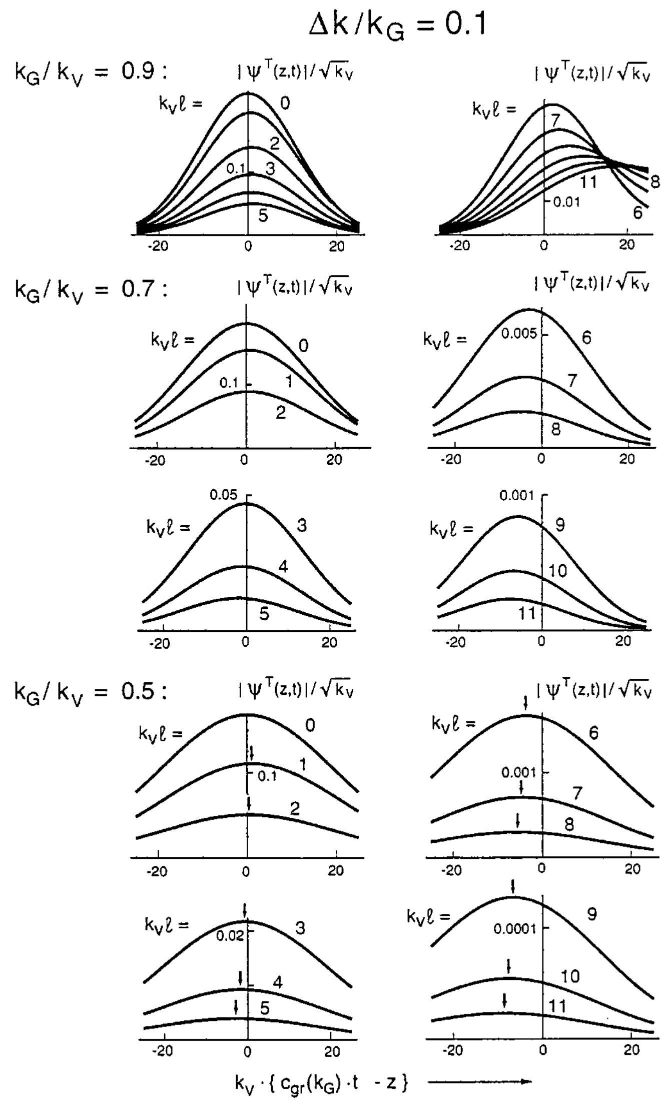

Fig. 4. Absolute value of the wave function of the transmitted wave packet $|\psi^T(z,t)|$ as a function of the common travelling wave argument $k_V(c_\mathrm{gr}(k_\mathrm{G})t - z)$. All chosen parameters are indicated. The incident Gaussian wave packet is indicated by $k_V\ell = 0$. The barrier is characterized by $k_V\ell > 0$ and inserted somewhere to the left at the negative z axis. The shape distortion with rising barrier thickness is mainly visible as a reduction in amplitude and shift of the peak. The peak positions, derived from the method of stationary phase, are indicated by arrows for the case $k_\mathrm{G}/k_V = 0.5$.

we are free from pulse broadening due to dispersion for times Δt which gives here in the worst case ($k_G/k_V = 0.5$) the condition $k_V c_{gr}(k_G)\Delta t \ll (k_G/\Delta k)^2 k_V/k_G = 200$. This corresponds to a dispersionless propagation at least about ten times the interval used in Figure 4, *i.e.*, the envelopes remain as shown at least in this time interval.

As expected, we obtain a strong reduction in amplitude by increasing the barrier thickness. Of course, this reduction is highest for the lowest kinetic energy ($k_G/k_V = 0.5$). For the highest kinetic energy ($k_G/k_V = 0.9$), most of the partial waves have a wave vector $k < k_V$, but a certain number have energies above the barrier and have $k > k_V$. A strong pulse distortion is the result, especially for thicker barriers.

Now, we concentrate our attention on the pulse distortion for $k_G/k_V = 0.7$ and 0.5. In both cases, the position of the peak is shifted with increasing barrier thickness first to the right (delayed) and then to the left (advanced). This is in accordance with Hartman's result. For a quantitative comparison, we deduce from (17, 18) the shift $\delta(\ )$ of the peak of the transmitted pulse relative to the free propagating Gaussian pulse

$$\delta(k_V\{c_{gr}(k)\cdot t - z\}) = k_V \frac{\partial}{\partial k}\varphi^T(k)\Big|_{k=k_G} = k_V\ell[F(k_G/k_V, k_V\ell) - 1]. \quad (28)$$

These peak positions of a nearly monoenergetic pulse of energy E_G are indicated by arrows in Figure 4 for the case $k_G/k_V = 0.5$. Their agreement with the peak positions of the calculated envelopes of the transmitted pulses is excellent despite of the finite line width $\Delta k/k_G = 0.1$. In addition, this agreement shows the reliability of the method of stationary phase to determine the peak positions.

At first glance, the transmitted wave packets for $k_G/k_V = 0.7$ and 0.5 seem to be Gaussian in shape, obviously due to the dominant Gaussian factor in the integrand of (27).

3 Impossibility of physical supervelocities in a symmetric rectangular barrier

In the following we will prove that no physical supervelocities can be involved. This will be done by constructing a special wave packet with a broad k spectrum which is extremely concentrated at $z = 0$ at the time $t = 0$ if no barrier is present. This wave field will be compared with the modified wave field near $z = 0$ that appears at the same time $t = 0$ if a barrier is inserted which the wave packet has passed. Here, the actual position of the barrier on the negative z axis has no influence since $\underline{T}(k)$ in (7) does not depend on this position. In the limit of an infinite broad k spectrum we obtain the pulse response function of the barrier which is $T(z)$ and we will show that $T(z) = 0$ for $z > 0$. If we consider an arbitrary incident wave packet $\psi(z,t)$, then the shape $\psi^T(z,t)$ after passing an inserted barrier is obtained according to (8) from the convolution of the incident wave packet with $T(z)$. Since $T(z) = 0$ for $z > 0$, it is clear from (8) that, if the incident wave packet is equal zero at $t = 0$ for $z > 0$ when propagating in free space, the wave packet $\psi^T(z,t)$ transmitted through the barrier is zero too for $z > 0$ at $t = 0$. The tunneling wave packet is therefore *not* advanced.

3.1 Short pulse using a broadband k spectrum

An incident Gaussian wave packet becomes narrower in space with increasing spread Δk. In order to investigate pulse propagation with pulses in real space that are as short as possible, we have to use a "white" k spectrum. The broadest k spectrum for a pulse traveling in $+z$ direction starts at $k = 0$ and extends in the limit to $k = +\infty$. To see the influence of the transition to the infinitely wide spectrum and to connect the treatment to those of the preceding section, we choose the following normalized half-sided Gaussian spectrum with $k_G = 0$ and width Δk

$$\underline{\psi}(k, t=0) = \begin{cases} 2\sqrt{\sqrt{\pi}/\Delta k}\,\mathrm{e}^{-\frac{1}{2}\left\{\frac{k}{\Delta k}\right\}^2} & \text{if } k \geq 0 \\ 0 & \text{else.} \end{cases} \quad (29)$$

According to (4, 5), the corresponding wave function of the incident wave packet is described by a definite integral which can be directly integrated with the help of the complementary error function erfc( )

$$\psi(z,t) = \frac{1}{\pi}\sqrt{\sqrt{\pi}/\Delta k}\int_0^\infty \mathrm{d}k\,\mathrm{e}^{-k^2/(\sqrt{2}\Delta k)^2}\mathrm{e}^{-\mathrm{i}\hbar k^2 t/2m}\mathrm{e}^{\mathrm{i}kz} \quad (30)$$

$$= \sqrt{1/\{2\sqrt{\pi}a\Delta k\}}\,\mathrm{e}^{-z^2/2a}\ \mathrm{erfc}(-\mathrm{i}z/\sqrt{2a}), \quad (31)$$

where

$$a = 1/\{\Delta k\}^2 + \mathrm{i}\hbar t/m, \quad (32)$$

$$\mathrm{erfc}(u) = 1 - \mathrm{erf}(u) = 2/\sqrt{\pi}\int_u^\infty \mathrm{d}v\,\exp(-v^2). \quad (33)$$

The absolute value $|\psi(z,t)|$ is sketched in Figure 5 for the times $t < 0$, $t = 0$, and $t > 0$, with a broad spectrum with finite spread Δk in the first column and with the limit $\Delta k \to \infty$ in the second column.

For $t = 0$, expression (31) simplifies to

$$\psi(z, t=0) = \sqrt{\Delta k/2\sqrt{\pi}}\,\mathrm{e}^{-(z\Delta k)^2/2}\mathrm{erfc}(-\mathrm{i}z\Delta k/\sqrt{2}). \quad (34)$$

The absolute value of this expression describes by its second factor a narrow Gaussian wave packet of spread $1/\Delta k$ in real space, centered at $z = 0$. However, this packet is symmetrically deformed and broadened due to the last factor, the complementary error function. Since the wave packet is normalized and becomes narrower with increasing Δk, we have a sharply peaked function at $z = 0$

$$T(z) = \int_{-\infty}^{\infty} \underline{T}(k) e^{ikz} dk/2\pi = \int_{-\infty}^{\infty} e^{ikz} dk/2\pi + \int_{-\infty}^{\infty} \{\underline{T}(k) - 1\} e^{ikz} dk/2\pi$$
$$= \delta(z) + \int_0^{\infty} \{\underline{T}(k) - 1\} e^{ikz} dk/2\pi + \int_0^{\infty} \{\underline{T}(-k) - 1\} e^{-ikz} dk/2\pi$$
$$= \delta(z) + \int_0^{\infty} \left[\{\underline{T}(k) - 1\} e^{ikz} + c.c.\right] dk/2\pi = \delta(z) + 2\mathrm{Re} \int_0^{\infty} \{\underline{T}(k) - 1\} e^{ikz} dk/2\pi. \quad (37)$$

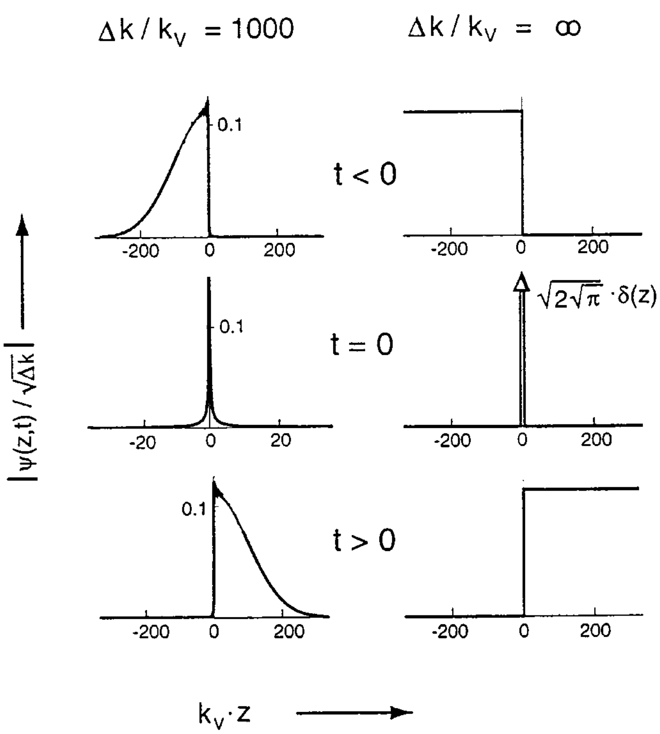

Fig. 5. Time behaviour of a wave packet with broadband k spectrum under the influence of dispersion during free one-dimensional propagation in the $+z$ direction. The chosen spectrum begins at $k = 0$, is of Gaussian shape, and has a spread Δk. In order to use dimensionless quantities, an arbitrary reference k_V is used. The three diagrams on the left correspond to a finite width $\Delta k = 1000\ k_V$ whereas the width is taken to be infinite on the right. At $t = 0$ the wave function is sharply peaked at $z = 0$, *i.e.* the electron will be localized there. The broader the spectrum, the more the wave function is totally restricted to the negative halfspace for $t < 0$ and to the positive halfspace for $t > 0$. Diagrams 1 and 3 on the left are calculated for times given by $k_V c_{\mathrm{gr}}(k_V)|t| = 0.05$.

at the time t =0 and may replace it in the limit $\Delta k \to \infty$ by

$$\left|\psi(z, t = 0)\sqrt{\Delta k}\right|_{\Delta k \to \infty} = \left|\Delta k/\sqrt[4]{\pi} e^{-(z\Delta k)^2/2}\right|_{\Delta k \to \infty} = \sqrt[4]{2\pi}\delta(z). \quad (35)$$

Hence, the position probability density narrows extremely at $t = 0$ to a Dirac delta function $\delta(z)$ at $z = 0$.

Since we treat only pulse propagation in the $+z$ direction and include dispersion which gives rise to group velocities proportional to the magnitude of the k vector, the sharply peaked wave packet at $t = 0$ was spread out for $t < 0$ mainly in the negative z region from about $z = -c_{\mathrm{gr}}(\Delta k)|t|$ to about $z = 0$ and for $t > 0$ mainly in the positive z region from about $z = 0$ to about $z = c_{\mathrm{gr}}(\Delta k)|t|$. The partial waves with small k value contribute mainly to the wave packet in the neighborhood of $z = 0$, whereas the higher values contribute to the more distant parts.

The expression for the transmitted wave packet becomes

$$\psi^T(z, t) = 1/\sqrt{\pi\sqrt{\pi}\Delta k} \int_0^{\infty} dk \underline{T}(k) \times e^{-k^2/(\sqrt{2}\Delta k)^2} e^{-i\hbar k^2 t/2m} e^{ikz}. \quad (36)$$

In the following discussion, it is more convenient to evaluate $\psi^T(z, t)$ by means of the convolution integral (8). This integral shows directly that in the case of an excitation in the form of a Dirac delta pulse $\psi(z) = const\ \delta(z)$ the transmitted wave packet is given by the *delta-pulse response (Green's function)* defined in (9)

see equation (37) above.

In the first line, the splitting of the integral takes account of the fact that for $|k| \gg |k_V|$ we have $\underline{T}(k) \approx 1$, *i.e.* the barrier does not disturb the propagation. This delta-pulse response (37) is plotted in Figure 6 using $k_V \ell = 0$, 1, 2, 4, 6, 8 as parameters. Whereas the first term in (37) reproduces the incident wave packet at $t = 0$, the second term in this delta-pulse response gives the deviations at $t = 0$ in z space due to the influence of the barrier. We remember that we have inserted the barrier at a sufficient large distance from $z = 0$ on the negative z axis in order to discuss the wave packet after the barrier. In this way, a direct comparison can be made in the neighborhood of $z = 0$ between the wave function without a barrier and the modified wave function after having passed the inserted barrier. It is important to note that if the second term in (37) delivers a contribution in the region $z > 0$, the transfer velocities of some partial plane waves must be higher than their group velocity, *i.e.* the insertion of a symmetrical barrier would cause supervelocities. Figure 6 reveals that we have at $t = 0$ for all five numerically investigated barriers no wave function in the region $z > 0$,

$$T(z) = \int_{-\infty}^{\infty} \underline{T}(k)\mathrm{e}^{\mathrm{i}kz}\mathrm{d}k/2\pi = \int_{-\infty}^{\infty} \frac{\mathrm{e}^{-\mathrm{i}k\ell}\mathrm{e}^{\mathrm{i}kz}}{\cosh\gamma\ell + \frac{\mathrm{i}}{2}\{\gamma/k - k/\gamma\}\sinh\gamma\ell}\frac{\mathrm{d}k}{2\pi} = \int_{-\infty}^{\infty} \frac{\mathrm{i}4k\gamma\mathrm{e}^{(\gamma-\mathrm{i}k)\ell}\mathrm{e}^{\mathrm{i}kz}}{(\gamma+\mathrm{i}k)^2 - (\gamma-\mathrm{i}k)^2\mathrm{e}^{2\gamma\ell}}\frac{\mathrm{d}k}{2\pi}$$
$$= \int_{-\infty}^{\infty} \frac{\mathrm{i}2k\gamma\mathrm{e}^{(\gamma-\mathrm{i}k)\ell}\mathrm{e}^{\mathrm{i}kz}}{(\gamma+\mathrm{i}k)^2}\left\{\frac{1}{1-\mathrm{e}^{\gamma\ell}(\gamma-\mathrm{i}k)^2/k_V^2} + \frac{1}{1+\mathrm{e}^{\gamma\ell}(\gamma-\mathrm{i}k)^2/k_V^2}\right\}\frac{\mathrm{d}k}{2\pi} \quad (38)$$

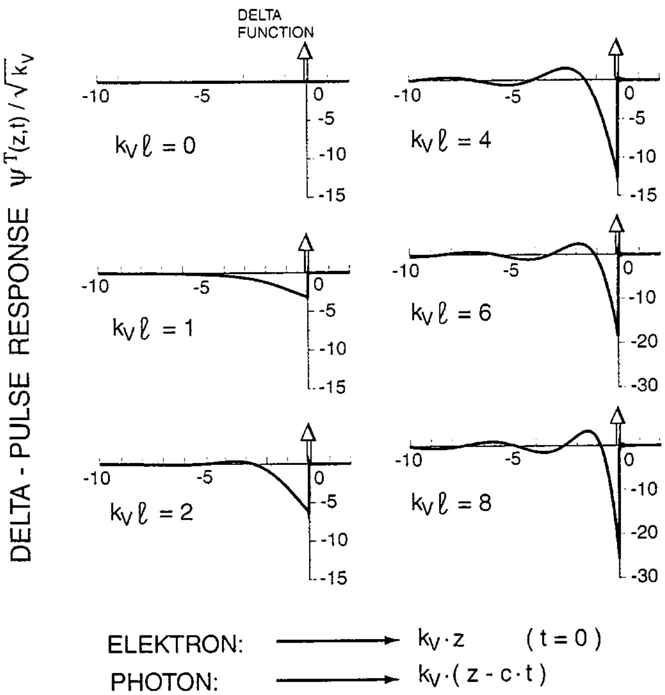

Fig. 6. The incident wave packet of an electron in vacuum reaches at $t = 0$ the origin $z = 0$ in the form of a Dirac-delta pulse (case $k_V\ell = 0$). If a barrier, fully described by $k_V\ell > 0$, is inserted on the left of the shown abscissa, the transmitted wave function takes at $t = 0$ the spatial dependence shown in the corresponding diagrams. In the case of photons, where no dispersion occurs, the wave function shown travels to the right with the velocity of light.

i.e. no supervelocities occur. This statement, based on the numerical investigation, will be proven to be general in the next section by an analytic investigation of the second term.

In the dispersionless case of photon tunneling across the vacuum gap between two prisms, the simplification consists only in the fact that the incident pulse $\delta(z - ct)$ and also the transmitted wave packet do not change their envelopes during propagation. Since $T(z)$ is the same, we have to conclude that in this arrangement no superluminal velocities occur as well.

3.2 Analytic proof that no supervelocities occur during transmission through a symmetric barrier

According to the results of the last section, we have to prove analytically that $T(z > 0) = 0$ for all times $t \leq 0$. To this purpose, we use contour integration in the complex $k = k' + \mathrm{i}k''$ plane. The integral

see equation (38) above

can be closed by a semicircle in the infinity of the upper halfspace $k'' > 0$ since the argument vanishes there for $z > 0$ with sufficient strength. As Cauchy has shown, this contour integral gives only contributions at the enclosed poles of the integrand according to $2\pi\mathrm{i}$ times the sum of their residues. However, we will show now that there are no poles at all in the upper halfspace. This yields the wanted result: $T(z) = 0$ if $t \leq 0$ and $z > 0$.

First we notice that there are no poles of the integrand at the real k axis since $|\underline{T}(k)|$ is according to (15) always greater than or equal to unity. In the complex k plane, the poles of the integrand of (38) must fulfill the condition (with $n = 0$ or $\pm 1, \pm 2, ...$)

$$\mathrm{e}^{\mathrm{i}k_V\sqrt{k^2/k_V^2-1}\ell/2} = 1/\left\{k/k_V - \sqrt{k^2/k_V^2 - 1}\right\}\mathrm{e}^{\mathrm{i}n\pi/2} = \left\{k/k_V + \sqrt{k^2/k_V^2 - 1}\right\}\mathrm{e}^{\mathrm{i}n\pi/2}, \quad (39)$$

where we have used expression (12) for γ. If k_ρ is the position of a pole then $-k_\rho^*$ also fulfills relation (39), *i.e.* poles always lie symmetric to the imaginary axis or on it. Taking the natural logarithm of (39) gives

$$\mathrm{i}k_V\sqrt{k^2/k_V^2 - 1}\ell/2 = \ln\{k/k_V + \sqrt{k^2/k_V^2 - 1}\} + \mathrm{i}n\pi/2$$

and thereafter taking the real part of this expression, we obtain the condition

$$-\mathrm{Im}\left\{\sqrt{k^2/k_V^2 - 1}\right\} = \frac{2}{k_V\ell}\ln\left|k/k_V + \sqrt{k^2/k_V^2 - 1}\right|. \quad (40)$$

The term on the right is outside of the real axis always greater than zero. Therefore, we may also write the condition for poles

$$\mathrm{Im}\left\{\sqrt{k^2/k_V^2 - 1}\right\} < 0. \quad (41)$$

Since $\sqrt{k^2/k_V^2 - 1}$ maps the content of the upper k/k_V half plane again onto the upper half plane, condition (41) is not fulfilled in the upper half plane. Hence, the upper half plane is free of poles. It is this statement we wanted to prove.

There are an infinite number of poles in the lower k/k_V half plane.

4 Distortion of a Gaussian voltage pulse by inserting a parallel resonance circuit into a transmission line

We have seen that the distortion of a Gaussian wave packet in crossing a barrier is responsible for the incorrectly claimed supervelocities. In the following, we want to show that an analogous distortion with corresponding "supervelocities" appears also in a transmission line with inserted concentrated *LCR*-circuit elements, *i.e.* there is no need for a barrier with associated exponential field decay or tunneling to demonstrate the Hartman effect.

In this connection, we have to mention the pure time-domain investigations of Mitchell and Chiao [24]. They demonstrated with very low frequency bandpass amplifiers that a Gaussian input pulse may yield a similar output pulse which precedes with its peak that of the input pulse. The amplification in this active system masks that both the phase shift and the signal attenuation of the *LCR* filters used are essential for this remarkable signal distortion. We hope that this aspect becomes more evident in the following discussion of our chosen passive system.

We consider an infinite homogeneous lossless transmission line which may be a coaxial line, a Lecher line or any other double-conductor line with lateral dimensions small in comparison to the wavelength. The characteristic line impedance is denoted by Z_0 (see Fig. 7). The extension ℓ' of this circuit is always taken equal zero since we consider a concentrated *LCR* parallel resonance circuit with a position in one of the conductors anywhere between $z = -\ell$ and $z = 0$. However, other circuits can also be used. Some examples are a parallel *CR* circuit in one conductor, a *LCR* series resonance circuit between both conductors, and more complicated circuits.

We assume that an incident Gaussian voltage pulse with center frequency ω_G, spread Δt, peak value u_G, and propagation velocity equal to the velocity of light c travels to the right:

$$u(z,t) = u_G e^{-(t-z/c)^2/2\Delta t^2} \cos\{\omega_G(t - z/c)\}. \qquad (42)$$

Hence, if *no LCR-circuit is inserted*, the voltage at position $z = 0$ becomes

$$u(z=0,t) = u_G e^{-t^2/2\Delta t^2} \cos \omega_G t \qquad (43)$$

with the associated Fourier spectrum

$$\underline{u}(z=0,\omega) = \sqrt{\frac{\pi}{2}}\frac{u_G}{\Delta\omega}\left\{e^{-(\omega-\omega_G)^2/2\Delta\omega^2} + e^{-(\omega+\omega_G)^2/2\Delta\omega^2}\right\} \qquad (44)$$

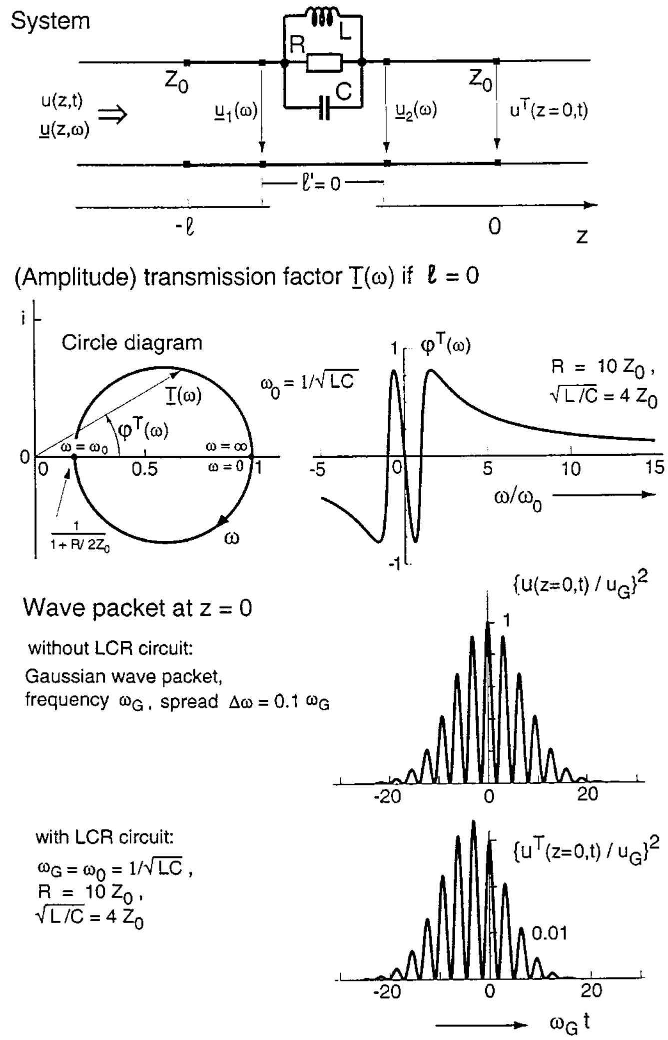

Fig. 7. Double-conductor transmission line with parallel *LCR* resonance circuit inserted into one of the conductors. The transmission factor $\underline{T}(\omega)$ describes in the complex plane a circle as a function of frequency for the case $\ell = 0$, *i.e.* with the circuit at $z = 0$. This gives rise to a phase dependence on frequency $\varphi^T(\omega)$ with positive slope in a frequency band containing the resonance frequency ω_0. Therefore, choosing appropriate circuit parameters and an incident Gaussian wave packet centred at ω_0, a negative time shift of the peak of the transmitted wave packet in comparison with the peak of the incident wave packet should occur. The two diagrams of the calculated wave packets show indeed that the peak of the transmitted wave packet leaves earlier the position of the circuit at $z = 0$ than the peak of the incoming Gaussian wave packet arrives. Please note the different vertical scales in these two diagrams.

where $\Delta\omega = 1/\Delta t$.

The *insertion of the resonance circuit at* $z = 0$ gives rise to a (amplitude) reflection factor $\underline{R}(\omega) = \{Z_0 - \underline{z}(\omega)\}/\{Z_0 + \underline{z}(\omega)\}$ where the impedance $\underline{z}(\omega)$ is given by the impedance $\underline{z}'(\omega)$ of the resonance circuit in series with the input impedance Z_0 of the following line. The Fourier component $\underline{u}_1(\omega)$ just before the resonance circuit is given by the superposition of the incident and reflected waves

$$\underline{u}_1(\omega) = \underline{u}(z=0,\omega)\{1+\underline{R}(\omega)\} = \underline{u}(z=0,\omega)/\{1+\underline{z}'(\omega)/(2Z_0)\}. \quad (45)$$

The Fourier component $\underline{u}_2(\omega) = \underline{u}_1(\omega)Z_0/\{Z_0+\underline{z}'(\omega)\}$ just after the resonance circuit relative to the incident Fourier component $\underline{u}(z=0,\omega)$ defines the (amplitude) transmission factor of a line junction of length $\ell = 0$ containing the resonance circuit

$$\underline{T}(\omega) \equiv |\underline{T}(\omega)|\mathrm{e}^{\mathrm{i}\varphi^T(\omega)} = \frac{\underline{u}_2(\omega)}{\underline{u}(z=0,\omega)} = \frac{1+\mathrm{i}2R/\sqrt{L/C}\{\omega/\omega_0-\omega_0/\omega\}}{1+R/(2Z_0)+\mathrm{i}2R/\sqrt{L/C}\{\omega/\omega_0-\omega_0/\omega\}}, \quad (46)$$

where we have used the expressions derived earlier and the resonance frequency $\omega_0 = 1/\sqrt{LC}$. The time dependent voltage $u^T(z=0,t) = u_2(t)$ just behind the inserted resonance circuit follows by Fourier transforming equation (46)

$$u^T(z=0,t) = \int_{-\infty}^{\infty} \underline{T}(\omega)\underline{u}(z=0,\omega)\frac{\mathrm{d}\omega}{2\pi} = \frac{1}{\sqrt{2\pi}}\frac{u_\mathrm{G}}{\Delta\omega}\int_0^{\infty}\{\mathrm{e}^{-(\omega-\omega_\mathrm{G})^2/2\Delta\omega^2}+\mathrm{e}^{-(\omega+\omega_\mathrm{G})^2/2\Delta\omega^2}\}|\underline{T}(\omega)| \times \cos\{\varphi^T(\omega)+\omega t\}\mathrm{d}\omega, \quad (47)$$

where we have used (44) in the last line. The peak position of the transmitted wave packet (47) occurs at $z=0$ according to the method of stationary phase at the time

$$t_2 = -\partial\varphi^T(\omega)/\partial\omega \quad \text{at} \quad \omega=\omega_\mathrm{G}. \quad (48)$$

Depending on the sign of $\partial\varphi^T(\omega)/\partial\omega$, the start time t_2 of the peak of the transmitted wave packet at $z=0$ after the inserted resonance circuit may be earlier or later in comparison with the arrival time $t=0$ of the peak without the inserted LCR circuit which is equal to the arrival time $t_1=0$ at $z=0$ just in front of the resonance circuit. Thus, we have to ask: Does a frequency region exist where $\partial\varphi^T(\omega)/\partial\omega$ is positive? This would mean according to (48) that the transmitted voltage peak behind the resonance circuit leaves the position $z=0$ earlier than the peak of the incident wave packet arrives at the circuit at $z=0$. To answer this question, we look at the plot of $\underline{T}(\omega)$ as a function of frequency in the complex plane. One obtains a circle (see Fig. 7) starting at $\underline{T}(\omega=0)=1$ and ending at the same value for $\omega=\infty$. With rising frequency, the phase change $\partial\varphi^T(\omega)/\partial\omega$ is negative at first, becomes positive around the resonance frequency ω_0 and thereafter becomes negative again. This is also visible in the plot of $\varphi^T(\omega)$ calculated for the noted special values of the resonance-circuit elements. We learn from these plots that an incident Gaussian packet with center frequency $\omega_\mathrm{G}=\omega_0$ should show this peculiar effect. This can also be shown by numerical integration of the integral (47). One obtains the diagram of the transmitted wave packet $\{u^T(z=0,t)/u_\mathrm{G}\}^2$ in Figure 7 which can be directly compared with the incident wave packet $\{u(z=0,t)/u_\mathrm{G}\}^2$ at the same position without the inserted resonance circuit. We observe indeed that *the peak of the transmitted wave packet leaves the position* $z=0$ *earlier, i.e. at* $t_2<0$, *than the peak of the incident Gaussian packet arrives at* $t_1=0$. Of course, the transmitted packet has a strongly reduced peak amplitude which guarantees causality, *i.e.* that, at any instant, the transmitted energy can fully be taken from the already arrived energy.

In the last step of our present discussion, we consider a *finite line section* between $z=-\ell$ and $z=0$ as shown in Figure 7. The travelling time across this section without the resonance circuit is ℓ/c. If we insert anywhere in this section the resonance circuit and if we take according to Hartman the two peaks as reference, the transmission time of this section becomes $\ell/c+t_2$. Hence, we obtain a transmission velocity

$$c_\mathrm{Ha} = \ell/(\ell/c+t_2) = c/(1+t_2c/\ell) = c/\{1-\partial\varphi^T(\omega)/\mathrm{d}\omega c/\ell\} \quad (49)$$

which becomes in regions of a finite negative t_2 or positive $\partial\varphi^T(\omega)/d\omega$, respectively, a superluminal velocity of arbitrary high magnitude depending only on the chosen length ℓ. Moreover negative transmission velocities c_Ha are possible.

The result of this example has again demonstrated that Hartman's method of defining the transmission times is unacceptable in physics.

5 Summary and conclusion

We studied in detail analytically as well as numerically the transmission of an incident Gaussian wave packet through a symmetric rectangular barrier for electrons and for light. In the following, we summarize the main results and conclusions of this paper.

(1) The shape of the transmitted wave packet was calculated by numerical integration for several sets of parameters. The peak positions were compared with the predictions from the method of stationary phase. As expected, both are in full agreement with one another. It is known from the literature that they are also in agreement with the results of electromagnetic wave-packet experiments.

(2) Hartman defined his transmission time τ_Ha by taking the time difference between the appearance of the peak of the transmitted wave packet after the barrier and the arrival of the peak of the incident Gaussian wave packet at the front of the barrier. The transmission velocity c_Ha derived from τ_Ha and the barrier thickness yields under certain conditions supervelocities, *i.e.* velocities greater than the electron velocity or velocity of light in vacuum, respectively.

(3) By considering the propagation of a broadband pulse with the limiting case of an infinite bandwidth, we

were able to prove in the present paper that the insertion of a symmetric rectangular barrier cannot produce a supervelocity. Hence, the method of Hartman to relate the two peaks of the wave packets has no justification in physics since they are not causally connected.

(4) The peak of the transmitted wave packet is formed by distortion of the incident wave packet due to strong amplitude reduction in combination with phase changes. This phenomenon is not restricted to barriers with their exponentially decaying near fields, but occurs also in other systems. This has been demonstrated in the last chapter using a transmission line with an LCR-resonance circuit inserted. Here it was shown that the peak of the transmitted packet can leave the circuit before the peak of the incident Gaussian wave packet arrives at the circuit. By considering a finite line section with inserted LCR circuit, the phenomenon of supervelocities in the sense of Hartman also appeared. This discussion revealed once more the physical irrelevance of the "transmission time" τ_{Ha} and of the "transmission velocity" c_{Ha}.

(5) We have to conclude that wave packets are in general unsuitable to determine transmission times through barriers.

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